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Adaptive Sliding-Mode Guidance of a Homing Missile

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Sliding-mode control is applied to design robust homing missile guidance law. The sufficient and necessary condition for the sliding-mode motion of a linear time-varying system not to be affected by disturbances and the sufficient condition for that motion not to be affected by parameter perturbations are given. An adaptive reaching law of sliding mode for a linear time-varying system is then presented and used to derive an adaptive sliding-mode guidance law. Theoretical analysis and simulation results show that the adaptive sliding-mode guidance law is robust against disturbances and parameter perturbations. Furthermore, the presented guidance law is simple to implement in practice.

I. Introduction

FOR an application in practical guidance, a guidance law should be robust to uncertainties. Because sliding-mode control is robust to parameter perturbations and external disturbances,[1] applying sliding-mode control to design guidance law has been investigated. Brierley and Longchamp[2] applied a sliding-mode control law to a nonlinear system representing an air-air missile-target interception process. They proved that the performance of the feedback controller is robust to certain parameter variations in the model.[2] Babu et al.[3] proposed a new form of proportional navigation guidance law with an additive-switched bias term for short-range homing missiles by invoking sliding-mode control theory.[3,4]

This work deals with an application of sliding-mode control to the guidance of homing missiles for achieving a robust guidance law. The sufficient and necessary condition for the sliding-mode motion of a linear time-varying system not to be affected by disturbances and the sufficient condition for that motion not to be affected by parameter perturbations are presented in Sec. II. A linear time-varying model describing the missile-target engagement is provided in Sec. III. Based on the theory in Sec. II and the model in Sec. III, sliding-mode control is used to derive an adaptive sliding-mode guidance law (ASMG) in Sec. IV. Simulation results are provided to show that the ASMG is robust to uncertainties in Sec. V. Finally, the conclusions are presented in Sec. VI.

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II. Invariance of Sliding Mode to Disturbances and Parameter Perturbations

A sliding-mode control system is well known to be robust to certain disturbances and parameter perturbations. In this section we will study the conditions for the sliding-mode motion of a linear time-varying system not to be disturbed by the two uncertainties.

Consider a linear time-varying system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} \qquad \boldsymbol{x} \in R^n, \quad \boldsymbol{u} \in R^m \tag{1}$$

where $\boldsymbol{x}$ represents the system state, $\boldsymbol{u}$ represents the control vector, t represents time, and $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ are time-varying matrices with appropriate dimensions. Let the switching function be

$$\boldsymbol{s} = \boldsymbol{C}(t)\boldsymbol{x} \qquad \boldsymbol{s} \in R^m \tag{2}$$

The sliding mode is in the subspace of $\boldsymbol{s} = 0$. In the following part of this section, we will discuss the behaviors of the sliding-mode system (1–2) under disturbances and parameter perturbations.

A. Invariance to Disturbances

Consider system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} + \boldsymbol{D}(t)\boldsymbol{f} \qquad \boldsymbol{x} \in R^n, \quad \boldsymbol{u} \in R^m, \quad \boldsymbol{f} \in R^l \tag{3a}$$

$$\boldsymbol{s} = \boldsymbol{C}(t)\boldsymbol{x} \qquad \boldsymbol{s} \in R^m \tag{3b}$$

where $\boldsymbol{f}$ represents a $l \times 1$ disturbance vector and $\boldsymbol{D}(t)$ is a $n \times l$ time-varying matrix.

The following theorem indicates the behavior of the sliding mode under the disturbance vector $\boldsymbol{f}$.

Theorem 1: The sliding mode of system (3) would not be affected by any disturbance if and only if

$$\text{rank}[\boldsymbol{B}(t), \boldsymbol{D}(t)] = \text{rank}[\boldsymbol{B}(t)]$$

To prove Theorem 1, we need a lemma.

Letting $\dot{s} = 0$ with system (3) induces

$$[\dot{\boldsymbol{C}}(t) + \boldsymbol{C}(t)\boldsymbol{A}(t)]x + \boldsymbol{C}(t)\boldsymbol{B}(t)\boldsymbol{u} + \boldsymbol{C}(t)\boldsymbol{D}(t)\boldsymbol{f} = 0 \tag{4}$$

Assume that $m \times m$ matrix $\boldsymbol{C}(t)\boldsymbol{B}(t)$ is invertible; thus, the equivalent control

$$\boldsymbol{u} = -[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}[\dot{\boldsymbol{C}}(t) + \boldsymbol{C}(t)\boldsymbol{A}(t)]\boldsymbol{x} - [\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\boldsymbol{D}(t)\boldsymbol{f} \tag{5}$$

can be solved from Eq. (4).

Substituting Eq. (5) into Eq. (3a) gives the differential equation of the sliding-mode motion

$$\dot{\boldsymbol{x}} = \{\boldsymbol{A}(t) - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}[\dot{\boldsymbol{C}}(t) + \boldsymbol{C}(t)\boldsymbol{A}(t)]\}\boldsymbol{x} + \{\boldsymbol{I} - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\}\boldsymbol{D}(t)\boldsymbol{f} \tag{6a}$$

Moreover, $\boldsymbol{s} = 0$ is

$$\boldsymbol{C}(t)\boldsymbol{x} = 0 \tag{6b}$$

With Eq. (6a), it is enough to obtain the following lemma.

Lemma 1: The sliding-mode motion of system (3) would not be affected by any disturbance if and only if

$$\{\boldsymbol{I} - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\}\boldsymbol{D}(t)\boldsymbol{f} = 0 \tag{7}$$

Next, Theorem 1 is to be proved.

Proof:

1) Necessity:

Rewrite Eq. (7) as

$$\boldsymbol{D}(t)\boldsymbol{f} = \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\boldsymbol{D}(t)\boldsymbol{f} \tag{8}$$

which indicates that any linear combination of the columns of matrix $\boldsymbol{D}(t)$ must be the linear combination of the columns of matrix $\boldsymbol{B}(t)$, i.e.,

$$\text{rank}[\boldsymbol{B}(t), \boldsymbol{D}(t)] = \text{rank}[\boldsymbol{B}(t)]$$

2) Sufficiency:

Because

$$\text{rank}[\boldsymbol{B}(t), \boldsymbol{D}(t)] = \text{rank}[\boldsymbol{B}(t)]$$

we have

$$\boldsymbol{D}(t)\boldsymbol{f} = \boldsymbol{B}(t)\boldsymbol{m}(t)$$

where $\boldsymbol{m}(t)$ is a m-dimensional column vector.

Under this condition, it follows that

$$\boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\boldsymbol{D}(t)f = \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\boldsymbol{B}(t)\boldsymbol{m}(t) = \boldsymbol{B}(t)\boldsymbol{m}(t) = \boldsymbol{D}(t)\boldsymbol{f}$$

Therefore, with the results in 1 and 2, it is easy to find that

$$\text{rank}[\boldsymbol{B}(t), \boldsymbol{D}(t)] = \text{rank}[\boldsymbol{B}(t)] \Leftrightarrow \{\boldsymbol{I} - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\}\boldsymbol{D}(t)\boldsymbol{f} = 0$$

Now, for Lemma 1, Theorem 1 has been proved.

B. Invariance to Parameter Perturbations

Consider system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \Delta\boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} \qquad \boldsymbol{x} \in R^n, \quad \boldsymbol{u} \in R^m \tag{9a}$$

$$\boldsymbol{s} = \boldsymbol{C}(t)\boldsymbol{x} \qquad \boldsymbol{s} \in R^m, \quad m < n \tag{9b}$$

The following theorem indicates the behavior of the sliding mode under a parameter perturbation matrix $\Delta\boldsymbol{A}(t)$.

Theorem 2: The sliding mode of system (9) would not be affected by any parameter perturbation if

$$\text{rank}[\boldsymbol{B}(t), \Delta\boldsymbol{A}\boldsymbol{t}_i] = \text{rank}[\boldsymbol{B}(t)] \qquad i = 1, 2, \ldots, n-m$$

where $\boldsymbol{t}_i$ is the column vector of a $n \times (n-m)$ matrix $\boldsymbol{T}(t)$, which constitutes the base of subspace $\boldsymbol{s}_0 = \ker\boldsymbol{C}(t)$, i.e., $\boldsymbol{C}(t)\boldsymbol{T}(t) = 0$.

Proof:

The equation of the sliding-mode motion of system (9) is

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \Delta\boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} \qquad \boldsymbol{C}(t)\boldsymbol{x} = 0$$

Solving $\dot{s} = 0$, i.e.,

$$\dot{\boldsymbol{C}}(t)\boldsymbol{x} + \boldsymbol{C}(t)\boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{C}(t)\Delta\boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{C}(t)\boldsymbol{B}(t)\boldsymbol{u} = 0$$

we have the equivalent control

$$\boldsymbol{u} = -[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}[\dot{\boldsymbol{C}}(t) + \boldsymbol{C}(t)\boldsymbol{A}(t) + \boldsymbol{C}(t)\Delta\boldsymbol{A}(t)]\boldsymbol{x}$$

Here, the $m \times m$ matrix $\boldsymbol{C}(t)\boldsymbol{B}(t)$ is assumed to be invertible. Thus, the equation of sliding-mode motion is

$$\dot{\boldsymbol{x}} = \{\boldsymbol{A}(t) - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}[\dot{\boldsymbol{C}}(t) + \boldsymbol{C}(t)\boldsymbol{A}(t)]\}\boldsymbol{x} + \{\boldsymbol{I} - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\}\Delta\boldsymbol{A}(t)\boldsymbol{x} \tag{10a}$$

$$\boldsymbol{C}(t)\boldsymbol{x} = 0 \tag{10b}$$

Apparently, if

$$\{\boldsymbol{I} - \boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\}\Delta\boldsymbol{A}(t)\boldsymbol{x} = 0 \tag{11}$$

then $\Delta\boldsymbol{A}(t)$ will not disturb the sliding-mode motion.

When $\boldsymbol{x} \in \boldsymbol{s}_0 = \mathit{ker}\,\boldsymbol{C}(t)$, there exists a $n-m$ dimensional column vector α to make

$$\boldsymbol{x} = \boldsymbol{T}(t)\alpha, \qquad \boldsymbol{T}(t) = [\boldsymbol{t}_1(t) \quad \cdots \quad \boldsymbol{t}_{n-m}(t)]$$

where $\boldsymbol{T}(t)$ satisfies $\boldsymbol{C}(t)\boldsymbol{T}(t)=0$ and $\boldsymbol{t}_1(t),\ldots,\boldsymbol{t}_{n-m}(t)$ are linearly independent n dimensional column vectors.

Because

$$\mathrm{rank}[\boldsymbol{B}(t),\,\Delta\boldsymbol{A}(t)\boldsymbol{t}_i(t)]=\mathrm{rank}[\boldsymbol{B}(t)]\qquad i=1,2,\ldots,n-m$$

i.e.,

$$\mathrm{rank}[\boldsymbol{B}(t),\,\Delta\boldsymbol{A}(t)\boldsymbol{T}(t)]=\mathrm{rank}[\boldsymbol{B}(t)]$$

gives

$$\Delta\boldsymbol{A}(t)\boldsymbol{T}(t)\alpha=\boldsymbol{B}(t)\boldsymbol{m}(t)$$

where $\boldsymbol{m}(t)$ is a m-dimensional column vector, we obtain

$$\begin{aligned}&\boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\Delta\boldsymbol{A}(t)\boldsymbol{T}(t)\alpha\\&\quad=\boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\boldsymbol{B}(t)\boldsymbol{m}(t)=\boldsymbol{B}(t)\boldsymbol{m}(t)\\&\quad=\Delta\boldsymbol{A}(t)\boldsymbol{T}(t)\alpha\end{aligned}$$

that is,

$$\{\boldsymbol{I}-\boldsymbol{B}(t)[\boldsymbol{C}(t)\boldsymbol{B}(t)]^{-1}\boldsymbol{C}(t)\}\Delta\boldsymbol{A}(t)\boldsymbol{T}(t)\boldsymbol{\alpha}(t)=0\qquad(12)$$

Equation (12) is equivalent to Eq. (11); therefore,

$$\mathrm{rank}[\boldsymbol{B}(t),\,\Delta\boldsymbol{A}(t)\boldsymbol{t}_i(t)]=\mathrm{rank}[\boldsymbol{B}(t)]\qquad i=1,2,\ldots,n-m$$

is sufficient for the sliding-mode motion not to be disturbed by $\Delta\boldsymbol{A}(t)$.

Now, Theorem 2 has been proved.

In the last part of this section, we consider a canonical form system

$$\dot{x}_i=x_{i+1}\qquad i=1,2,\ldots,n-1\qquad(13a)$$

$$\dot{x}_n=-\boldsymbol{a}^T(t)x-\Delta\boldsymbol{a}^T(t)x+b(t)u+d(t)f\qquad(13b)$$

where $\boldsymbol{a}(t)$ and $\Delta\boldsymbol{a}(t)$ are n-dimensional column vectors and $\Delta\boldsymbol{a}(t)$ represents the perturbation of $\boldsymbol{a}(t)$. Verifying that the canonical form system naturally satisfies the invariance conditions of sliding mode to disturbances and parameter perturbations is easy.

III. Formulation of Missile-Target Engagement

To describe the missile-target engagement, we choose a line of sight (LOS) coordinate system ($ox_3y_3z_3$) as the reference coordinate system where the origin o is taken to be the current mass center of the missile and the x_3 axis is taken to be the initial LOS. The y_3 axis is perpendicular to the x_3 and positive upward. The z_3 axis should be determined by the right-hand rule. During the interception process, define the elevation angle and azimuth angle with reference to $ox_3y_3z_3$; then both of them are small variables. Moreover, the roll angle of the missile is zeroed by the missile's attitude control system, so we may assume that the elevation loop and the azimuth loop are not coupled with each other. In the following discussion we need to study the target-to-missile motion in only one of the loops, for example, the elevation loop because the motion in the azimuth loop is similar to that in the elevation loop.

Assume that $q(t)$ represents the elevation angle, $r(t)$ represents the target-to-missile range, and $r_{y_3}(t)$ represents the component of $r(t)$ in axis y_3.

Because $q(t)$ is a small variable, we have

$$q(t)=\frac{r_{y_3}(t)}{r(t)}\qquad(14)$$

Differentiating Eq. (14) twice with respect to time produces

$$\ddot{q}(t)=-a_1(t)q(t)-a_2(t)\dot{q}(t)+b(t)\ddot{r}_{y_3}(t)\qquad(15)$$

where

$$a_1(t)=\ddot{r}(t)/r(t),\qquad a_2(t)=2\dot{r}(t)/r(t),\qquad b(t)=1/r(t)\qquad(16)$$

$$\ddot{r}_{y_3}(t)=-a_M(t)+a_T(t)\qquad(17)$$

In Eq. (17) a_M and $a_T(t)$ represent the missile's acceleration and the target's acceleration in axis y_3, respectively.

Substituting Eq. (17) into Eq. (15) gives

$$\ddot{q}(t)=-a_1(t)q(t)-a_2(t)\dot{q}(t)-b(t)a_M(t)+b(t)a_T(t)$$

Let $x_1=q(t)$ and $x_2=\dot{q}(t)$. The first-order linear time-varying differential equation group used to describe the missile-target engagement takes the form of

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix}=\begin{bmatrix}0&1\\-a_1(t)&-a_2(t)\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\-b(t)\end{bmatrix}u+\begin{bmatrix}0\\b(t)\end{bmatrix}f\qquad(18)$$

where $u=a_M(t)$ and $f=a_T(t)$ are respectively the control variable and the disturbance in system (18).

IV. ASMG

Based on the theory in Sec. II, system (18) is obviously a canonical form system and is robust to disturbances and parameter perturbations. In fact, $r(t)$ and $\dot{r}(t)$, which determine the two parameters $a_1(t)$ and $a_2(t)$, cannot be measured or estimated accurately, so these are two perturbation parameters. If the target performs a maneuver normal to the LOS, then its acceleration constitutes a disturbance. In short, both disturbances and parameter perturbations exist in system (18) inevitably, so that designing a robust guidance law with sliding-mode control theory is very significant.

Based on the heuristic that zeroing the LOS angular rate must eventually lead to intercept, we choose the switching function as

$$s=r(t)\dot{q}(t)\qquad(19)$$

Integrating Eq. (19) with system (18), we have

$$s=r(t)x_2\qquad(20)$$

Using a reaching law of sliding mode to synthesize a sliding-mode control system can achieve good dynamical performance.[1] In general, a reaching law is described by the following first-order differential equation:

$$\begin{aligned}&\dot{s}=-f(s)-\varepsilon\,\mathrm{sgn}\,(s),\qquad\varepsilon>0\\&f(0)=0,\qquad sf(s)>0\qquad\text{if}\quad s\neq0\end{aligned}\qquad(21)$$

where $f(s)$ denotes a function about s.

For a constant system or a slow time-varying system, satisfactory dynamical performance may be achieved by using an exponential reaching law, i.e.,

$$\begin{aligned}&\dot{s}=-ks-\varepsilon\,\mathrm{sgn}\,s\qquad k>0,\qquad\varepsilon>0,\\&k=\mathrm{const},\qquad\varepsilon=\mathrm{const}\end{aligned}\qquad(22)$$

System (18) is a fast time-varying system, and so Eq. (22) is ineffective. Especially, when $r(t)\to0$, the inertia of a constant reaching law will make the LOS angular rate diverge, and this phenomenon will amplify the miss distance. Therefore, an adaptive reaching law must be considered. Its general form is

$$\begin{aligned}&\dot{s}=-f(s,p)-\varepsilon(p)\,\mathrm{sgn}\,(s),\qquad\varepsilon(p)>0\\&f(0,p)=0,\qquad sf(s,p)>0\qquad\text{if}\quad s\neq0\end{aligned}\qquad(23)$$

where p represents the parameters in the sliding-mode system.

We choose

$$\dot{s} = -\frac{k|\dot{r}(t)|}{r(t)}s - \varepsilon \operatorname{sgn} s$$
$$k = \text{const} > 0, \qquad \varepsilon = \text{const} > 0 \tag{24}$$

as the adaptive reaching law. The physical meaning of Eq. (24) is that the speed of tending to the sliding mode is adjusted with the target-to-missile range $r(t)$. Especially when $r(t)$ is near to zero, the speed arises rapidly to prevent $\dot{q}(t)$ from diverging. This reaching law is also useful for alleviating chattering.

Differentiating Eq. (20) and considering system (18) give

$$\dot{s} = \dot{r}(t)x_2 + r(t)\dot{x}_2 = -\dot{r}(t)x_2 - \ddot{r}(t)x_1 - u + f \tag{25}$$

where $r(t) > 0$ and $\dot{r}(t) < 0$.

Substituting Eq. (25) and Eq. (20) into Eq. (24) gives

$$u = (k+1)|\dot{r}(t)|x_2 - \ddot{r}(t)x_1 + \varepsilon \operatorname{sgn} x_2 + f \tag{26}$$

which is an exact expression of the ASMG.

In fact, the target's acceleration is unknown and is viewed as a disturbance, so a practical ASMG should be

$$u = (k+1)|\dot{r}(t)|x_2 - \ddot{r}(t)x_1 + \varepsilon \operatorname{sgn} x_2 \tag{27}$$

To implement Eq. (27) precisely, $\dot{r}(t)$ and $\ddot{r}(t)$ should be known. In practice, the sliding-mode control system is robust to parameter perturbations, so that $\dot{r}(t)$ and $\ddot{r}(t)$ need not be estimated on line, but may be assumed to be constants, i.e., $\dot{r}(t) \approx \hat{\dot{r}}(0) = \text{const}$, and $\ddot{r}(t) \approx 0$, where $\hat{\dot{r}}(0)$ represents the initial estimate value of relative speed.

Furthermore, to alleviate chattering, the variable structure item $\varepsilon \operatorname{sgn} x_2$ can be replaced with a continuous function $\varepsilon x_2/(|x_2| + \delta)$, where δ is a small positive real number.

Thus, a simplified ASMG is written as

$$u = (k+1)|\hat{\dot{r}}(0)|x_2 + \varepsilon x_2/(|x_2| + \delta) \tag{28}$$

Equation (28) shows that the ASMG is improved relative to proportional navigation (PN) and is simple to implement in practice. The effective navigation ratio $k+1$ is usually $3 \sim 5$.

By the second method of Lyapunov, let the Lyapunov function be $V = \frac{1}{2}x_2^2$. Differentiating V with respect to time and considering system (18) give

$$\dot{V} = x_2\left[-\frac{\ddot{r}(t)}{r(t)}x_1 - \frac{2\dot{r}(t)}{r(t)}x_2 - \frac{u}{r(t)} + \frac{f}{r(t)}\right] \tag{29}$$

Substituting Eq. (27) into Eq. (29) gives

$$\dot{V} = \frac{(k-1)\dot{r}(t)}{r(t)}x_2^2 - \frac{(\varepsilon \operatorname{sgn} x_2 + f)}{r(t)}x_2 \tag{30}$$

Equation (30) shows that when $k > 1$, if $\varepsilon > |f|$, $\dot{V} < 0$ is then sufficiently guaranteed. Assuming that $k > 1$, $\dot{r}(t) \le -\beta < 0$, $0 < r(t) \le M$, β and M are constants, and $\varepsilon > |f|$, then

$$\dot{V} = \frac{(k-1)\dot{r}(t)}{r(t)}x_2^2 - \frac{(\varepsilon \operatorname{sgn} x_2 + f)}{r(t)}x_2 \le -\frac{2(k-1)\beta}{M}V$$
$$\Rightarrow V(t) \le V_0 \exp[2(k-1)\beta/M]t \Rightarrow V(t) \to 0, \qquad t \to \infty$$

i.e., $x_2 = \dot{q} \to 0$

V. Simulation of Space Interception

In this section we apply the ASMG to the space interception problem. In the simulation the missile is modeled by the dynamical system with six degrees of freedom, and the target is modeled by the dynamical system with three degrees of freedom. The following coordinate systems are adopted:

Earth Center Inertia Coordinate System $o_e x_e y_e z_e$

The origin o_e is taken to be the center of the Earth. The vertical plane $o_e x_e y_e$ coincides with the translation plane of the target. The x_e axis directs to a star. The y_e axis is perpendicular to the x_e, and its positive direction directs upward. The target is in the first quadrant of plane $o_e x_e y_e$. The z_e axis is determined by the right-hand rule.

Inertia Reference Coordinate System $o'xyz$

For the whole intercept period the origin o' is taken to be the initial mass center of the missile. Axes x, y, and z are parallel to axes x_e, y_e, and z_e, respectively.

Missile Body Coordinate System $ox_1y_1z_1$

The origin o is taken to be the current mass center of the missile. The x_1 axis coincides with the missile longitudinal axis, and its positive direction directs to the nose of the missile. The y_1 axis is in the vertical plane of symmetry of the missile body, and its positive direction directs upward. The z_1 axis is determined by the right-hand rule.

Missile Translation Coordinate System $ox_2y_2z_2$

The origin o is taken to be the current mass center of the missile. The x_2 axis coincides with the direction of $\boldsymbol{\nu}_M$, the velocity of the missile mass center. The y_2 axis is in the vertical plane containing $\boldsymbol{\nu}_M$. It is perpendicular to the x_2 and positive upward. The z_2 axis is determined by the right-hand rule. Rotating $o'xyz$ to $ox_2y_2z_2$ produces the missile's vertical flight-path angle θ_M and horizontal flight-path angle φ_M. In the same way it is easy to set up the target translation coordinate system $o''x_2y_2z_2$ and define the target's vertical flight-path angle θ_T and horizontal flight-path angle φ_T, where the origin o'' is the current mass center of the target.

The missile's initial velocity and flight-path angles are

$$\boldsymbol{\nu}_{M0} = 3500\ \text{m/s}, \qquad \theta_{M0} = 50\ \text{deg}, \qquad \varphi_{M0} = -1.5\ \text{deg} \tag{31}$$

The missile's initial position in the coordinate system $o'xyz$ is

$$x_{M0} = 0\ \text{km}, \qquad y_{M0} = 0\ \text{km}, \qquad z_{M0} = 0\ \text{km} \tag{32}$$

The target's initial velocity and flight-path angles are

$$\boldsymbol{\nu}_{T0} = 7300\ \text{m/s}, \qquad \theta_{T0} = 10\ \text{deg}, \qquad \varphi_{T0} = 180\ \text{deg} \tag{33}$$

The target's initial position in $o'xyz$ is

$$x_{T0} = 100\ \text{km}, \qquad y_{T0} = 15\ \text{km}, \qquad z_{T0} = 1\ \text{km} \tag{34}$$

In the interception process the LOS angular rates are measured by an infrared seeker installed in the front of the missile. A microcomputer on the missile inputs the LOS angular rates and calculates guidance commands by guidance law. The sampling period of the microcomputer is 20 ms. Translation thrusters are then controlled to apply forces according to the guidance command. Around the mass center of the missile are located four independently controlled thrusters. Two of them are aligned with the positive and negative direction of the horizontal body axis y_1. Another two are aligned with the positive and negative direction of the vertical body axis z_1.

Assume that the control force is a constant over a sampling period. The effect of force in every sampling period can be achieved by controlling the opening time and the closing time of the proper thruster. The maximum translation acceleration of the missile in both the elevation loop and the azimuth loop are 10 m/s^2.

A. PN

First, the simulation results for a conventional guidance method, PN, are given and will form the baseline for comparison purposes. For this case

$$u = K|\dot{r}(t)|x_2 \tag{35}$$

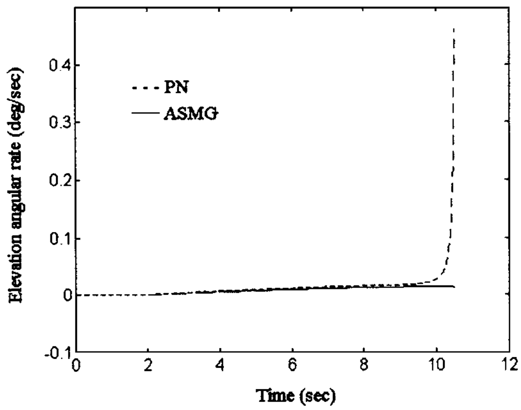

Fig. 1 Elevation angular rate.

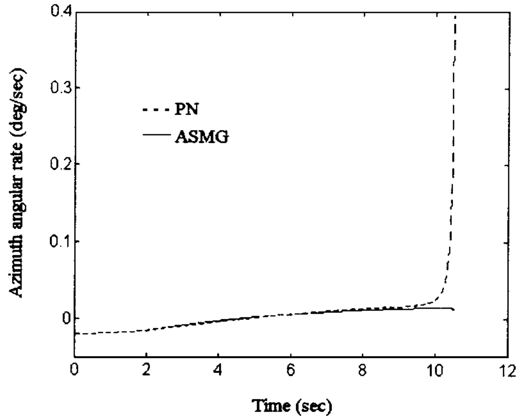

Fig. 2 Azimuth angular rate.

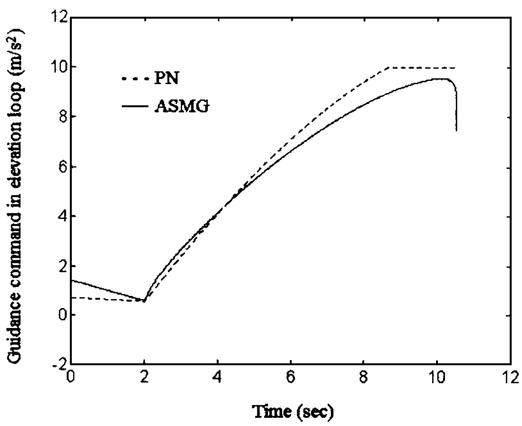

Fig. 3 Guidance command in the elevation loop.

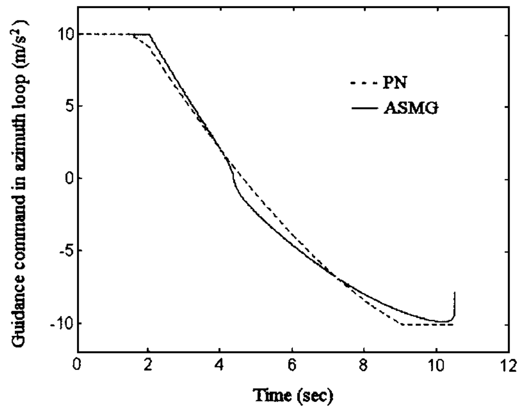

Fig. 4 Guidance command in the azimuth loop.

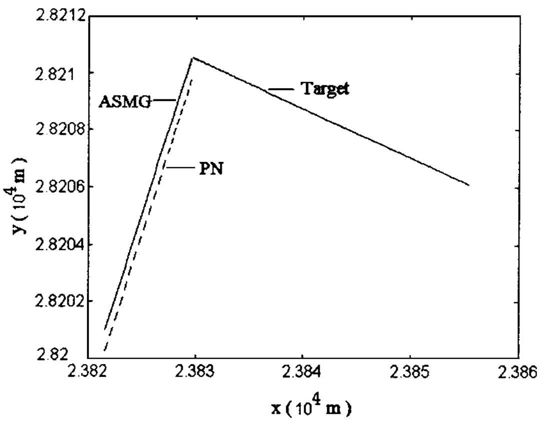

Fig. 5 Eventual flight path in the *xy* plane.

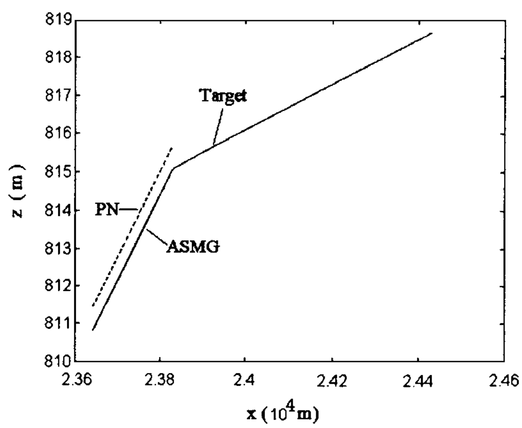

Fig. 6 Eventual flight path in the *xz* plane.

The effective navigation ratio is usually chosen as $K = 3 \sim 5$. Here we let $K = 4$ and use an initial estimate value $\hat{\dot{r}}(0) = -8558$ m/s to replace $\dot{r}(t)$ because the infrared seeker cannot measure $\dot{r}(t)$. The target begins a maneuver at $t = 2$ s with $a_{T_{y_2}} = 5$ m/s^2 and $a_{T_{z_2}} = 5$ m/s^2 where $a_{T_{y_2}}$ and $a_{T_{z_2}}$ are the components of target acceleration in the target translation coordinate system $o''x_2y_2z_2$. The maneuver lasts for all of the subsequent time of the interception process.

The variations of the elevation angular rate and the azimuth angular rate under the PN are respectively shown in Figs. 1 and 2. The results show that the LOS angular rates will ultimately diverge because the target maneuver saturates the PN's guidance commands in both elevation loop and azimuth loop (see the dotted lines in Figs. 3 and 4). The divergence of LOS angular rates amplify the final miss distance, which can be clearly seen from the eventual flight path of the missile in Figs. 5 and 6 (see the dotted lines). In fact, the final miss distance of PN is 0.94 m.

B. ASMG

Next, we demonstrate the performance of the ASMG with $k = 3$ and $\varepsilon = 1$. Also let the initial estimation value of the relative speed be $\hat{\dot{r}}(0) = -8558$ m/s. The real value of $\dot{r}(0)$ is -9544 m/s. This estimation error will not produce apparent impact on guidance results, because the ASMG is robust to parameter perturbations. The target's maneuver begins at $t = 2$ s with $a_{T_{y_2}} = 5$ m/s^2 and $a_{T_{z_2}} = 5$ m/s^2 and lasts for all of the subsequent time of the interception process.

In theory, if ε is chosen to be larger than $|f|$, then $\dot{q}(t)$ tends to zero. In practice, precise guidance requires that $\dot{q}(t)$ lie in a region that is sufficiently near zero while the guidance is terminated for the missile's seeker stepping into its blind area. For this reason we let ε be an appropriate positive value less than $|f|$, for instance, let $\varepsilon = 1$; then $\dot{q}(t)$ is not nullified but restricted in a mini-neighboring region of zero. Because $\dot{q}(t)$ does not have to remain at zero, no chattering occurs.

The variations of the elevation angular rate and the azimuth angular rate under the ASMG are also respectively plotted in Figs. 1

and 2 to compare with the results of PN. With the effect of ε term, the LOS angular rates are restricted in a small neighborhood of zero before the guidance is terminated, so that the missile is guided so precisely that the final miss distance is only 0.001 m, which is hardly seen from the missile's flight path in Figs. 5 and 6 (see the solid lines). Moreover, no chattering is visible from the variations of LOS angular rates under the ASMG for the application of the adaptive reaching law and the appropriate choice of ε. This result also can be seen from the variations of the ASMG's guidance commands in both elevation loop and azimuth loop (see Figs. 3 and 4). No frequent switches are needed in the guidance commands; therefore, no chattering occurs. Another fact in Figs. 3 and 4 is that the target maneuver has not saturated the ASMG's guidance commands; this ensures the stability of the LOS angular rates.

VI. Conclusions

The adaptive sliding-mode guidance law has good performance in intercepting a maneuvering target because it is robust against disturbances and parameter perturbations. Moreover, this guidance law is apt to be implemented in practice for its ability to eliminate chattering and its simple form.

Acknowledgment

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References

[1] Gao, W. B., *Fundamentals of Variable Structure Control Theory,* 1st ed., Chinese Science and Technology Press, Beijing, 1990, pp. 24–37.

[2] Brierley, S. D., and Longchamp, R., "Application of Sliding-Mode Control to Air-Air Interception Problem," *IEEE Transactions on Aerospace and Electronic Systems,* Vol. 26, No. 2, 1990, pp. 306–325.

[3] Babu, K. R., Sarma, I. G., and Swamy, K. N., "Switched Bias Proportional Navigation for Homing Guidance Against Highly Maneuvering Target," *Journal of Guidance, Control, and Dynamics,* Vol. 17, No. 6, 1994, pp. 1357–1363.

[4] Babu, K. R., Sarma, I. G., and Swamy, K. N., "Two Variable-Structure Homing Guidance Schemes With and Without Target Maneuver Estimation," AIAA Paper 94-3566, Aug. 1994.